MTH 253
Final Review

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1. Determine whether the sequence $\{\tan n\}$ converges or diverges.
2. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+1)!}{2^n}$ converges or diverges. Justify your conclusion as specifically as possible.
3. Write the first five terms of the sequence $\left\{\dfrac{n(n+1)(2n+1)}{6}\right\}$.
4. Graph the first five terms of the sequence $\left\{\dfrac{n(n+1)(2n+1)}{6}\right\}$. Then graph the first five partial sums.
5. Consider the sequence $\{a_n\} = \{128, -96, 72, -54, 40.5, \ldots\}$.
   (a) What kind of a sequence is this?
   (b) Write a formula for a_n, the nth term of the sequence.
   (c) Determine whether the sequence $\{a_n\}$ converges or diverges. If the sequence converges, what does it converge to?
   (d) Consider the series $\displaystyle\sum_{n=1}^{\infty} a_n$. Find the first three partial sums.
   (e) Find the exact value of the sum of the series $\displaystyle\sum_{n=1}^{\infty} a_n$.
6. Determine whether the series below converges or diverges. Justify your conclusion as specifically as possible.
$$1 - \frac{1}{2} + 1 - \frac{1}{4} + 1 - \frac{1}{8} + 1 - \frac{1}{16} + \cdots$$
7. Determine whether the series below converges or diverges. Justify your conclusion as specifically as possible.
$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$
8. Consider the function $f(x) = \dfrac{x^3}{x^3+2}$.
   (a) Express $f(x)$ as a power series.
   (b) Determine the radius of convergence for the power series you found in (a).
   (c) Determine the interval of convergence for the power series you found in (a).
   (d) Find $f'(x)$ by differentiating the power series you found in (a) term-by-term.

(e) Find an antiderivative for f by integrating the power series you found in (a) term-by-term.

9. Consider the power series below.

$$\sum_{n=0}^{\infty} \frac{(x+2)^n}{n4^n}$$

Find the radius and interval of convergence for the power series.

10. Let $g(x) = \cosh x = \frac{1}{2}\left(e^x - e^{-x}\right)$.
    (a) Find a Maclaurin series for $g(x)$. Write out at least the first four nonzero terms of the series. Then write your final conclusion in sigma notation.
    (b) What is the interval of convergence for the power series you found in part (a)?
    (c) Find a Taylor series for $g(x)$ centered at $a = \ln 2$. Write out at least the first four nonzero terms of the series. Do *not* write your final conclusion in sigma notation.
    (d) What is the interval of convergence for the power series you found in part (c)?
    (e) Find the third-degree polynomial $T_3(x)$ for $g(x)$ centered at $a = \ln 2$.

11. Let $f(x) = \sqrt[5]{(1+x)^4}$.
    (a) Use the Binomial Series to expand $f(x)$ as a power series. Write the first four terms of the series in the expanded form as well as the summation notation.
    (b) What is the radius of convergence for the power series you found in part (a)?

12. Let $g(x) = \dfrac{2}{(1+x)^4}$.
    (a) Use the Binomial Series to expand $g(x)$ as a power series. Write the first four terms of the series in the expanded form as well as the summation notation.
    (b) What is the radius of convergence for the power series you found in part (a)?