MTH 252 Lab
Area Between Curves

Damien Adams

Purpose

Integration is a vastly useful tool. We will be able to use integration to compute anything that can be represented as an accumulation. We have seen now that a definite integral represents the net area between the x-axis and the curve of $y = f(x)$ between $x = a$ and $x = b$. Now we have an integral that represents the area between two curves.

(a) What is the integral that represents the area of the region bounded above by $y = f(x)$, below by $y = g(x)$, to the left by $x = a$, and to the right by $x = b$?

(b) What is the integral that represents the area of the region bounded above by $y = d$, below by $y = c$, to the left by $x = g(y)$, and to the right by $x = f(y)$?

Prompts

1. Let $f(x) = 2^x$ and $g(x) = -x^2 + 2x + 1$.
   a. Use Desmos to graph both $f(x)$ and $g(x)$.
   b. Using the graph, identify the two points of intersection for these two curves. Determine which curve is greater than the other between the intersection points.
   c. Set up an integral that represents the area enclosed by the curves $y = f(x)$ and $y = g(x)$.
   d. Find the exact value of the area between the two curves.

2. Find the exact value of the area between the curves $y = \sqrt{x}$ and $y = x^3$.

3. On Desmos, graph both $x + y^2 = 56$ and $x + y = 0$. Identify the region enclosed by these two curves. Determine whether to integrate with respect to x or y, and find the area of the region.

4. Consider the curves given by $y = \sin x$ and $y = \cos x$. For each of the following problems, you should include a sketch of the region/solid being considered, as well as a labeled representative slice.
   (a) Sketch the region $\mathcal{R}$ bounded by the y-axis, $y = \cos x$, and $y = \sin x$ up to the first positive value of x at which the curves intersect. What is the exact intersection point of the curves? Be sure to list the *point*, not just an x- or y-value.
   (b) Set up a definite integral with differential dx whose value is the exact area of $\mathcal{R}$.
   (c) Set up a definite integral with differential dy whose value is the exact area of $\mathcal{R}$.

5. Consider the region $\mathcal{R}$ bounded by $y = \sin(x^2)$, $y = 0$, $x = 0$, and $x = \sqrt{\pi}$. Sketch this region in Desmos, and then set up (but do not evaluate) an integral that represents the exact area of $\mathcal{R}$.

6. Consider the region $\mathcal{S}$ bounded by $y = \sin(x^2)$, $y = 1$, and $x = 0$. Sketch this region in Desmos, and then set up (but do not evaluate) an integral that represents the exact area of $\mathcal{S}$.