MTH 252 Lab
Definite Integrals & Average Values

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Purpose

We are now into the thick of it with integrals – we're looking at definite integrals in addition to the previous indefinite integrals. In the last section, we saw $\sum_{i=1}^{n} f(x_i^*)\Delta x$ for different x_i^*. In this section, we take this Riemann sum and look at $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$ and shift our focus to $\int_a^b f(x)\ dx$.

(a) What is the difference between $\sum_{i=1}^{n} f(x_i^*)\Delta x$ and $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$?

(b) Geometrically, what does $\int_a^b f(x)\ dx$ represent?

(c) What is the formula for the average value of f on the interval $[a, b]$?

Prompts

1. Let $f(x) = \sqrt{64 - x^2}$.
   (a) Graph f in Desmos.
   (b) Evaluate $\int_0^8 \sqrt{64 - x^2}\ dx$.
   (c) Find the average value of f on $[0, 8]$.
   (d) Evaluate $\int_{-8}^8 \sqrt{64 - x^2}\ dx$.
   (e) Find the average value of f on $[-8, 8]$.
   (f) Evaluate $\int_{-8}^0 \sqrt{64 - x^2}\ dx$.
   (g) Find the average value of f on $[-8, 0]$.

2. Let $f(x) = |5 - x|$ and $g(x) = 2x - 2$.

   (a) Evaluate $\displaystyle\int_0^8 f(x)\ dx$.

   (b) Find the average value of f on $[0, 8]$.

   (c) Evaluate $\displaystyle\int_{-2}^4 g(x)\ dx$.

   (d) Find the average value of g on $[-2, 4]$.

3. Let $f(x) = 2x^2 + 3x - 4$.

   (a) Write a definite integral to represent the area between the x-axis and the graph of $y = f(x)$ between $x = -2$ and $x = 1$.

   (b) Write the integral you found in (a) as the limit of a Riemann sum.

   (c) Write an expression for Δx (the only letter in this expression should be n).

   (d) Choosing sample points to be right endpoints, we have that $x_i^* = x_i$. Write a formula for x_i for this particular Riemann sum (the only letters in this expressions should be i and n).

   (e) Evaluate the limit of this Riemann sum by first substituting in the expressions you found in (c) and (d) into (b), then expanding the summand, and then using your summation and limit properties.