MTH 252 Lab
Optimization

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Purpose

One of the most useful applications of differential calculus is optimization.

(a) Unless we have additional information, we use calculus (typically the Closed Interval Method) to identify the extrema of a function that is used to model a situation.

(b) We begin with a search for critical values, but we *do not stop there*.

(c) Don't forget to include a domain restriction if you are using the Closed Interval Method (as the name states, you need a closed interval).

Strategy for Optimization

1. Understand the situation, and draw a picture.
2. Define the variables.
3. Decide what we want to **optimize**, and write an equation that models the quantity.
4. If there is a **constraint**, write an equation that models the constraint.
5. Substitute the **constraint** into the **optimizing equation** to create a single-variable function that represents the quantity we want to optimize.
6. Determine if there is a domain restriction for which the optimizing function is valid.
7. Use Calculus (the Closed Interval Method, First-Derivative Test, and/or Second-Derivative Test) to optimize the function. Make sure to verify that your conclusion is inside the valid domain.
8. State your conclusion.

Closed Interval Method

To find the global extrema of a continuous function on $[a, b]$,

1. Find the critical numbers of f in (a, b).
2. Evaluate f at each critical value, c.
3. Evaluate f at each of the endpoints, a and b.
4. The largest of the values you found is the global maximum. The smallest is the global minimum.

Prompts

1. It is high noon (12pm), and a horde of zombies has been on its way for some time. Though those that knew about it were able to get a message to you:

   "It's too late for us, but you still have a chance! I was able to observe how many zombies were in each part of the horde as they approach you. It turns out

   $$z(t) = 10(2t^3 - 15t^2 + 24t + 20)$$

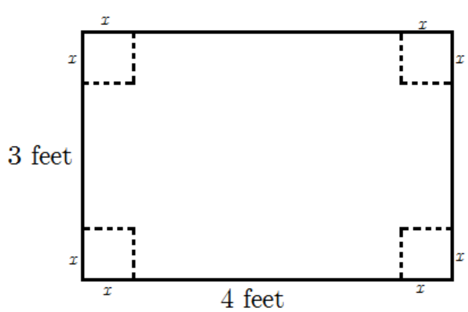
   represents the number of zombies that will be passing by you in hours after noon. You must get out of there no later than nightfall – 6pm. Good luck!"

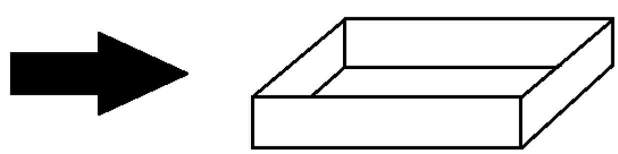
   This may be sobering, but this brave mathematician gave us a chance. We need to escape this classroom and get to the cafeteria to get supplies!

   Determine the best time to escape this room! *In this prompt, you do not need to find an optimizing equation or constraint – the equation you need, $z(t)$, is provided.*

2. You've successfully made it to a safe room. You need to take as many supplies with you as possible. The room will not hold for more than a day, so you need to get out and make a break for it.

   First, you should build some boxes to take these supplies. There are only a few pieces of cardboard, and each one is 3 feet wide by 4 feet long. If you cut squares out of the corners of each piece of cardboard, you can fold up and tape the sides, creating a box with an open top.

   What size squares should be cut out of the corners in order to maximize the volume of each box? What is the volume of the largest box that can be made?

3. Way to go! You not only got all the supplies you need, but you got out of the safe room to some farmland bordering a nearby creek! You are tired and need to recover. As luck would have it, there is an abandoned farmhouse on the property! There are plenty more supplies inside, and some seeds to start gardening. This is a perfect place to try to survive long-term.

   Inside the farmhouse, you found enough fencing materials for 2,000 feet of fence. You want to fence off a rectangular field along the river. We have not seen any evidence that zombies can swim, so we will not need to build any fence along the creek. It would probably be best to assume that the creek is straight.

   What are the dimensions of the field that has the largest area?

4. There is a light at the end of this tunnel – an extraction team is on the way and is planning to meet you 8 miles downstream on the other side of the creek. There is a canoe inside the farmhouse that can hold all of you. You must row across the 2-mile wide creek (it's a big creek) and get 8 miles downstream.

   You have a few options. You could row directly across the river and run 8 miles down to the rendezvous, you could row directly to the rendezvous, or you could row diagonally to some point partway to the rendezvous, x miles downstream, and run the remaining distance.

   If you can row at 6 mph and run at 8 mph, how far downstream should you row in order to get to the rendezvous as soon as possible? If the zombies will get to you in 1 hour and 15 minutes, will you make it?