MTH 252 Lab
Volumes

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Purpose

Integration has now represented area under a curve, area between curves, and volume of a solid of revolution. Depending on the situation, different integrals may be useful (and some may not be useful). Draw a picture

(a) Write down the three different integrals that represent volume. Draw a solid of revolution for each case.

(b) If a solid of revolution has a washer cross section, describe when the shell method is necessary for finding the volume of this solid.

Prompts

1. Consider a solid of revolution with volume V. When should a disk method be used to find V? When should the method of cylindrical shells be used to find V? When should a washer method be used for finding V? Draw a solid to represent each of these three situations.

2. Let $\mathcal{R}$ be the region in the first quadrant enclosed by the curves $y = \sin x$, $y = \cos x$, and the y-axis. For each of the following prompts, you should include a sketch of the region/solid being considered, as well as a labeled typical disk/washer/shell. *You do not need to evaluate these integrals, but I would encourage you to do so outside of the lab.*

   (a) Let $\mathcal{S}_1$ be the solid obtained by rotating $\mathcal{R}$ about the x-axis. Write a definite integral that represents the volume of $\mathcal{S}_1$.

   (b) Let $\mathcal{S}_2$ be the solid obtained by rotating $\mathcal{R}$ about the y-axis. Write a definite integral that represents the volume of $\mathcal{S}_2$.

   (c) Let $\mathcal{S}_3$ be the solid obtained by rotating $\mathcal{R}$ about the line $y = 2$. Write a definite integral that represents the volume of $\mathcal{S}_3$.

   (d) Let $\mathcal{S}_4$ be the solid obtained by rotating $\mathcal{R}$ about the line $x = -1$. Write a definite integral that represents the volume of $\mathcal{S}_4$.

3. Let $f(x) = 9 - x^2$. Let $\mathcal{S}$ be the solid obtained by rotating the region enclosed by the x-axis and $y = f(x)$ about the axis $x = -3$.

   (a) Which method(s) may be used to compute the volume of $\mathcal{S}$: Disk Method, Washer Method, and/or Shell Method?

   (b) Set up an integral that represents the volume of $\mathcal{S}$.

   (c) Find the volume of $\mathcal{S}$.

4. Let $f(x) = x^2 + 2$ and $g(x) = 4 - x^2$, and let $\mathcal{R}$ represent the region enclosed between $y = f(x)$ and $y = g(x)$. Let $\mathcal{S}$ be the "ring" obtained by rotating $\mathcal{R}$ about the x-axis.

   (a) Set up an integral that represents the area of $\mathcal{R}$.

   (b) Find the area of $\mathcal{R}$.

   (c) Set up an integral that represents the volume of $\mathcal{S}$.

   (d) Find the value of $\mathcal{S}$.

5. Let T be the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$. Let V be the volume of the solid obtained by rotating T about the line $x = a$ with $a > 1$. Find V when

   (a) $a = 2$

   (b) $a = 3$

   (c) $a = 10$