

# MTH 252 Lab

## Arc Length & Work

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### Purpose

A definite integral can represent many quantities, and arc length and work are two more examples of this. We study work to provide a small link of how calculus can be useful in physics. We study arc length for a more specific reason – it turns out, as you progress through calculus, the arc length integral becomes the foundation for which Vector Calculus II is built. Yes, it is useful to find the length of an arc, but its uses are much more powerful.

- Write down an integral that represents the length of the arc  $x = g(y)$  between  $y = c$  and  $y = d$ .
- Write down an integral that represents the work done by a force  $x$  as it moves an object from  $x = a$  to  $x = b$ .
- What law do we use to find a function to integrate when determining the work required to stretch a spring?
- What two units do we use to measure work done by a force?

### Prompts

- The curve  $y^2 = x^3$  is called a semicubical parabola. Find the length of the arc of  $y^2 = x^3$  from the point  $(1, 1)$  to the point  $(4, 8)$ . Begin by writing an integral that represents this length, and then evaluate that integral.
- Let  $C$  be the portion of the curve  $y = \ln(\cos x)$  between  $x = 0$  and  $x = \frac{\pi}{3}$ .
  - Write an integral that represents the exact length of  $C$ .
  - Use the fact that  $\int \sec x \, dx = \ln|\sec x + \tan x| + C$  to evaluate the integral you found in part (a).
  - Use Desmos to graph  $y = \ln(\cos x)$ . Does your conclusion in part (b) seem reasonable?
- A particle moving on a line is located  $x$  meters to the right of a point on the line called the origin when a force of  $\frac{x}{x^2 + 2}$  Newtons acts on it. How much work is done by the force in moving the particle from  $x = 1$  m to  $x = 3$  m? Write your conclusion as a sentence, including units.
- A spring has a natural length of 20cm. If a 25 Newton force is required to keep it stretched to a length of 30cm, how much work is required to stretch it from 20cm to 25cm? Write your conclusion as a sentence, including units.

## Student Skills

There are lots of **types of errors**. When you get full credit on a prompt, this is validation that you understand that concept. When you miss a point on an exam or assignment, you have an opportunity to learn from it. Any prompt in which you receive comments, it is your opportunity to understand what went wrong and to learn from it. Take time to determine what sort of an error you made.

### Types of Errors:

- Computational Errors – These errors occur when you make some sort of a miscalculation.
  - You may have made a small mistake by not being careful enough (e.g.  $2 + 3 = 6$  would be an error for mistaking addition for multiplication). These are typically avoided by making sure you are rested and working in a more mindful state.
  - You may not have understood how to evaluate a particular quantity (e.g.  $\arctan(1) = 45$  would be an error for not understanding that arctangent does not output degrees). These are typically mended by going back to do a few examples and writing down the proper rule each time you do a prompt. This doesn't usually take much time to patch.
  - You may have taught yourself something that is not true (e.g.  $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$  would be an error because square roots do not distribute over addition). These are typically mended by taking time each time you do math to write out the proper rule and doing one or two examples. This typically takes a bit of time to patch because you first have to unlearn the improper knowledge before learning the proper method.
- Conceptual Errors – These errors occur when you fundamentally misunderstand a concept.
  - If you missed a concept in class, you might want to take time to go back and learn that topic. Watch videos, ask questions, take notes, go to office hours, and do more exercises.
  - If you mislearned a concept, then you might need to go through the unlearning process (like with Computation). Just like above, review material, and go through the process of relearning.

Most importantly, be sure to learn from your mistakes. Never just take a comment and move on – take that time to learn.