MTH 252 Lab
Improper Integrals

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Purpose

Many functions whose domain includes (a, ∞) contain a finite area underneath its curve. Moreover, some functions whose graph contains a vertical asymptote also contain a finite area under its curve. In order to find these areas, we adapt a definite integral to work for us by utilizing a limit. Doing so will allow us to use the Fundamental Theorem of Calculus Part II.

(a) How can we rewrite $\int_a^\infty f(x)\,dx$ using a limit?

(b) If $f(x)$ has a vertical asymptote at $x = 2$, how can we rewrite $\int_0^2 f(x)\,dx$ using a limit? (Don't forget to include a left- or right-handed limit if necessary)

(c) If $f(x)$ has a vertical asymptote at $x = 2$, how can we rewrite $\int_0^3 f(x)\,dx$ using a limit?

Prompts

1. Determine if $\int_1^\infty e^{-x}\,dx$ converges or diverges. If it converges, find what it converges to. Show all work, and use proper notation as you work through.

2. Determine if $\int_1^\infty \frac{2}{x^5}\,dx$ converges or diverges. If it converges, find what it converges to. Show all work, and use proper notation as you work through.

3. Determine if $\int_1^\infty \frac{2}{\sqrt[5]{x}}\,dx$ converges or diverges. If it converges, find what it converges to. Show all work, and use proper notation as you work through.

4. Determine if $\int_0^1 \frac{2}{x^5}\,dx$ converges or diverges. If it converges, find what it converges to. Show all work, and use proper notation as you work through.

5. Determine if $\int_0^1 \frac{2}{\sqrt[5]{x}}\,dx$ converges or diverges. If it converges, find what it converges to. Show all work, and use proper notation as you work through.