MTH 252 Lab
Partial Fraction Decomposition

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Purpose

Integrating a rational function is actually *not* done by undoing the quotient rule. Instead, we use a method called *partial fraction decomposition*. This method involves breaking up (decomposing) the fraction into smaller (partial) pieces. We then use substitution to integrate the smaller pieces.

(a) We use PFD to decompose our fraction, then we integrate things like $\frac{-2}{x+1}$ or $\frac{-2}{(x+1)^2}$. How do we integrate $\int \frac{-2}{x+1}$?

(b) How do we integrate $\int \frac{-2}{(x+1)^2}$?

(c) We can use the Heaviside Cover-up to integrate a rational function whose denominator factors into *distinct linear factors*.

(d) We use the method of solving a system of linear equations when our denominator has repeated factors.

Prompts

1. Evaluate $\int \frac{x}{x^2 + x - 2} \, dx$

2. Evaluate $\int \frac{5}{x^2 + 5x + 6} \, dx$

3. Evaluate $\int_0^1 \frac{2x - 1}{x^2 - 2x - 3} \, dx$

4. Evaluate $\int \frac{1}{x(x-1)(x-2)(x-3)(x-4)} \, dx$

5. Evaluate $\int \frac{x^2 + x + 1}{x^4 - x^2} \, dx$

6. Evaluate $\int \frac{x^3 - 2}{(x^2 - 1)^2} \, dx$

7. Evaluate $\int_0^1 \frac{x - 1}{x(2x + 1)^3(x - 2)} \, dx$

8. Evaluate $\int \frac{1}{x^2(2x - 1)^2} \, dx$