# APPLICATIONS OF TAYLOR POLYNOMIALS

## MTH 253 LECTURE NOTES

**Exploration:** Suppose $f$ is equal to the sum of its Taylor series at $a$. Then $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$. Its $n$th degree Taylor polynomial is

$$\begin{aligned} T_n(x) &= \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i \\ &= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n \end{aligned}$$

Since $f$ is the sum of its Taylor series, $T_n \longrightarrow f$ as $n \longrightarrow \infty$. Thus, $T_n(x) \approx f(x)$ whenever $x$ is near $a$.

**Definition**

If $f$ is differentiable at $a$, then $L(x) = T_1(x)$ is called the **Linearization** of $f$ at $a$. That is, $L(x) = f(a) + f'(a)(x-a)$.

**Example 1.** Find the linearization of $f(x) = e^x$ at 10.

**Exercise 1.** Find the linearization of $g(x) = \sin x$ at 0.

**Technology Exploration:** Use Desmos to graph $g(x) = \sin x$ and its linearization at 0. Then graph a general linearization of $g(x)$ for an arbitrary value of $a$. What is the relationship between a linearization and $g$?

**Example 2.** Approximate $f(x) = \sqrt{x}$ by a Taylor polynomial of degree 3 at $a = 4$. How accurate is this approximation when $3 \leq x \leq 5$. Confirm your answer in GeoGebra.

**Example 3.** Approximate $g(x) = \sin x$ by a Taylor polynomial of degree 5 at $a = 0$. For what values of $x$ is this approximation accurate within 0.00005?

**Technology Exploration:** Use GeoGebra to graph $g(x) = \sin x$ and $T_5(x)$ found above. Does the interval found make sense?

**Example 4.** Approximate $\int_0^1 \arctan x \; dx$ using a Maclaurin polynomial of degree 6 for $\int \arctan x \; dx$.

**Exercise 2.** Approximate $\int_0^1 e^{-x^2} \; dx$ using a third-degree Taylor polynomial for $\int e^{-x^2} \; dx$.