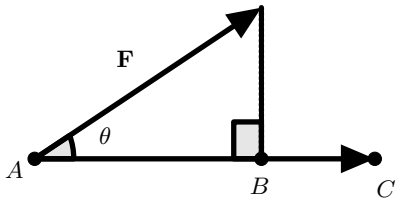
WORK

MTH 253 LECTURE NOTES

Exploration: Suppose a constant force is being exerted on an object to move it in a straight line. Then the force being exerted on the object has an *amount* of force as well as a *direction* of the force. So force can be represented by a vector! Suppose that the force, $\mathbf{F}$, is *not necessarily* being exerted in the same direction as the object is to move.

If the object moves from an initial point A to a terminal point C, then the displacement vector can be written as $\mathbf{D} = \overrightarrow{AC}$. The two vectors here are $\mathbf{D}$ and $\mathbf{F}$.



Notice that $\cos\theta = \dfrac{|\overrightarrow{AB}|}{|\mathbf{F}|}$, so $|\mathbf{F}|\cos\theta = |\overrightarrow{AB}|$. From physics, the work done by a vector $\mathbf{F}$ is defined as the product of the magnitude of the displacement, $|\mathbf{D}|$, and the magnitude of the applied force in the direction of the motion, $|\overrightarrow{AB}|$. In particular,

$$\begin{aligned} W &= |\mathbf{D}||\overrightarrow{AB}| \\ &= |\mathbf{D}|(|\mathbf{F}|\cos\theta) \\ &= |\mathbf{D}||\mathbf{F}|\cos\theta \\ &= \mathbf{D}\cdot\mathbf{F} \end{aligned}$$

Definition

The **Work** done by a force $\mathbf{F}$ on an object whose displacement vector is $\mathbf{D}$ is

$$W = \mathbf{F}\cdot\mathbf{D} = |\mathbf{F}||\mathbf{D}|\cos\theta$$

where θ is the angle between $\mathbf{F}$ and $\mathbf{D}$.

Units: Force is typically measured in pounds (lbs) (US) or Newtons (N) (metric).
Displacement is typically measured in feet (ft) (US) or meters (m) (metric).
Work is typically measured in foot-pounds (ft-lbs) (US) or Joules (J) (metric).

Example 1. A rolling backpack is being pulled a distance of 20 m along a horizontal path by a constant force of 50 N. The handle of the backpack is held at an angle of 55° above the horizontal. Find the work done by the force, rounded to the nearest Joule.

Exercise 1. A force given by a vector $\mathbf{F} = -2\mathbf{i}+4\mathbf{k}$ and moves a particle from $P(-1,-1,-1)$ to $Q(1,6,0)$. Find the work done by the force on the particle. Then find the angle between the force and displacement vectors.