MTH 255
Midterm Review

Damien Adams

1. Convert $(1, -2, 7)$ to cylindrical coordinates. Draw a three-dimensional coordinate system and plot this point. Round your values to the nearest hundredth.

2. Convert $(-3, -1, 2)$ to spherical coordinates. Draw a three-dimensional coordinate system and plot this point. Round your values to the nearest hundredth.

3. How can we express the plane $y = mx$ in spherical coordinates, where $m \in \mathbb{R}$?

4. Sketch the solid described by $\rho \leq 1$, $0 \leq \varphi \leq \frac{\pi}{6}$, and $0 \leq \theta \leq \pi$.

5. Find a parametrization for the part of the ellipsoid $x^2 + 4y^2 + 9z^2 = 16$ behind the yz-plane.

6. Find a parametrization for the part of the paraboloid $2x^2 + 2y^2 + z = 4$ in the first octant. If necessary, provide appropriate limitations on your parameters using inequalities that do not involve the other parameters.

7. Find the extrema of $f(x, y) = x^2 + y^2 + 9x - 9y$ subject to the constraint $x^2 + y^2 \leq 16$. Use the method of Lagrange multipliers.

8. Find the extrema of $h(x, y) = xy$ subject to the constraint $4x^2 + y^2 = 8$. Use the method of Lagrange multipliers.

9. Find the area of the part of the cylinder $x^2 + z^2 = 4$ that lies above the square with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$, and $(1, 1)$.

10. Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

11. Find the area of the part of the surface $z = 4 - 2x^2 + y$ that lies above the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.

12. Evaluate $\iiint_E y \, dV$, where $E = \{(x, y, z) \mid 0 \leq x \leq 3, 0 \leq y \leq x, x - y \leq z \leq x + y\}$.

13. Evaluate $\iiint_E \sin y \, dV$, where E lies below the plane $z = x$ and above the triangular region with vertices $(0, 0, 0)$, $(\pi, 0, 0)$, and $(0, \pi, 0)$.

14. Find the volume of the solid enclosed by the paraboloids $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$.

15. Evaluate $\iiint_E \sqrt{x^2 + y^2 + z^2} \, dV$, where E is the solid above the cone $z = \sqrt{x^2 + y^2}$ and between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

16. Find the volume of the solid above the cone $\varphi = \frac{\pi}{3}$ and beneath the sphere $\rho = 4 \cos \varphi$.

17. Find the Jacobian for the transformation $x = u + vw$, $y = v + wu$, and $z = w + uv$.

18. Consider $\iint_R (4x + 8y) \, dA$, where R is the parallelogram with vertices $(-1, 3)$, $(1, -3)$, $(3, -1)$, and $(1, 5)$. Evaluate the integral with the transformation $x = \frac{1}{4}(u + v)$, $y = \frac{1}{4}(v - 3u)$.

19. Evaluate $\iint_R (x + y)e^{x^2 - y^2} \, dA$, where R is the rectangle enclosed by $x - y = 0$, $x - y = 2$, $x + y = 0$, and $x + y = 3$.