The Leontief Input-Output model

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Introduction

This presentation seeks to explain and give an example of Linear Algebra as it applies to economics, using The Leontief Input-Output model.

Wassily Leontief

- Russian-American economist known for his development of the input output model
- Professor at Harvard University
- Developed for use with large economies
- Nobel Prize in Economic Sciences for his work on the Input-Output Model
- Model was expanded upon and widely used

Input-Output Model

	Industry 1	Industry 2	Industry n	Total Sales
Industry 1	x_{11}	x_{12}	x_{1n}	$X_1 = \sum_j x_{1j}$
Industry 2	x_{21}	x_{22}	x_{2n}	$X_2 = \sum_j x_{2j}$
...	...	...	...	...
Industry n	x_{n1}	x_{n2}	x_{nn}	$X_n = \sum_j x_{nj}$
Total Purchases	X_1	X_2	X_n	

Table: Input-Output Analysis in Economics

`http://www.economicsdiscussion.net/input-output-analysis/input-output-analysis-in-economics-economics/26833`

The Leontief Input Output Production Equation

- $\left\{ \begin{array}{c} \textit{Amount} \\ \textit{Produced} \\ \mathbf{x} \end{array} \right\} = \left\{ \begin{array}{c} \textit{Intermediate} \\ \textit{Demand} \\ C \end{array} \right\} + \left\{ \begin{array}{c} \textit{Final} \\ \textit{Demand} \\ \mathbf{d} \end{array} \right\}$

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- $\mathbf{x} = C\mathbf{x} + \mathbf{d}$

The Leontief Input Output Production Equation

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- $\mathbf{x} = C\mathbf{x} + \mathbf{d}$
- $(I - C)\mathbf{x} = \mathbf{d}$

Grocery Store Example

Department	Produce	Grocery	Bakery	Deli
Produce	Strawberries	Granola	Donut	Assembly
Grocery	Vegetables	Rice	Roll	Seasoned Meat
Bakery	Strawberries	Frosting	Cake Batter	Nothing
Deli	Lettuce	Mayo	Bread	Ham

This is what the 4x4 matrix will look like. Each department is only producing one good.

- Produce is Grab n Go breakfast packs
- Grocery is a ready-to-eat stir fry meal
- Bakery is producing cakes
- Deli produces sandwiches

Consumption Matrix

Department	Produce	Grocery	Bakery	Deli
Produce	.085	.039	.124	.201
Grocery	.012	.014	.071	.044
Bakery	.026	.308	.455	0
Deli	.014	.011	.027	.276

Formula for finding the matrix values: (Cost for producing \$1 of output)

- Contributing Cost = Amount Used x Cost
- Matrix Value = $\frac{Contributing Cost}{Price of Good}$

Solving the matrix

$$C = \begin{bmatrix} .085 & .039 & .124 & .201 \\ .012 & .014 & .071 & .044 \\ .026 & .308 & .455 & 0 \\ .014 & .011 & .027 & .276 \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} 10522 \\ 30780 \\ 3016 \\ 5576 \end{bmatrix} \quad (I - C) = \begin{bmatrix} .915 & -.039 & -.124 & -.201 \\ -.012 & .986 & -.071 & -.044 \\ -.026 & -.308 & .545 & 0 \\ -.014 & -.011 & -.027 & .724 \end{bmatrix}$$

- Augment and row reduce matrix $(I - C)$ with final demand vector $\mathbf{d}$ to find total amount produced $\mathbf{x}$
- Linear combination of matrix $(I - C)$ with amount produced vector $\mathbf{x}$ to find final demand vector $\mathbf{d}$

Solving the matrix

$$\left[\begin{array}{cccc|c} .915 & -.039 & -.124 & -.201 & 10522 \\ -.012 & .986 & -.071 & -.044 & 30780 \\ -.026 & -.308 & .545 & 0 & 3016 \\ -.014 & -.011 & -.027 & .724 & 5576 \end{array}\right]$$

$$RREF = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 18477.643952 \\ 0 & 1 & 0 & 0 & 33700.141997 \\ 0 & 0 & 1 & 0 & 25460.665097 \\ 0 & 0 & 0 & 1 & 9520.478639 \end{array}\right]$$

$$\mathbf{x} = \begin{bmatrix} 18477.64 \\ 33700.14 \\ 25460.67 \\ 9520.48 \end{bmatrix}$$

Input Output model uses

- Accounting
- Services
- Produced goods
- Environmental Economic Input-Output Model
- Forecasting

Questions?

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