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Introduction

Before we start factoring polynomials, it is good to understand the process of factoring and understand factoring of whole numbers. It turns out that knowing how to factor whole numbers will be very beneficial to factoring polynomials.

Factoring is the process of breaking a product into its factors.

Factors are the expressions multiplied together to form a product.

factor factor product

(2)(3) = 6

The numbers, 2 and 3 are factors and the 6 is a product.

Factoring completely is factoring a product down to a product of prime factors.

Factor 24 completely:

24
(2)(12)
(2)(2)(6)
(2)(2)(2)(3)

So 24 broken down to a product of primes is (2)(3)(2)(2).

A **prime factor** is a factor that has only one and itself as factors.

A **composite factor** has more than one and itself as factors.

The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, …

There are different techniques for factoring whole numbers. The following table shows the different methods.

Is this number a factor?	**Method to determine if a number is a factor of a particular number**
Is 2 a factor?	Two will be a factor if the number is an even number. An even number ends in 0, 2, 4, 6, or 8.
Is 3 a factor?	Three is a factor of a number if the digits of the number add up to a sum that is divisible by three. For example does the number 582 have 3 as a factor? Add up 5+8+2 and you get a sum of 15 which is divisible by 3, thus 582 does have 3 as a factor.
Is 4 a factor?	Four will be a factor of a number if the number formed by looking at the last two digits. If that number has four as a factor then the original number has four as a factor. For example the number 924 has four as a factor because in viewing the number formed by the last two digits, ***24***; the number 24 has four as a factor and the original number, 924, will also have four as a factor.

Is this number a factor?	**Method to determine if a number is a factor of a particular number**
Is 5 a factor?	Five will be a factor if the number ends in zero or five.
Is 6 a factor?	Six will be a factor is two and three are factors.
Is 9 a factor?	Nine will be a factor of a number when all of the digits add up to a sum that is divisible by 9. [Look at the method to see if three is a factor.]
Is 10 a factor?	Ten will be a factor if the number ends in zero.

Factoring Polynomials

When we factor polynomials, we will see there are different methods or techniques just as there are different techniques for factoring whole numbers.

Polynomials can also be factored into other polynomials. For example:

factor　factor　product

$$(x + 3)(x - 2) = x^2 + x - 6$$

(x + 3) is a factor and *(x – 2)* is a factor and the product is, $x^2 + x - 6$.

Polynomials are factored with different techniques based on the size of the polynomial. There are seven techniques or methods.

We will work with polynomials of 2, 3 and 4 terms.

Greatest Common Factor or GCF method should be tried on any size.

For a two term polynomial, try difference of squares or sum or difference of cubes.

For a three term polynomial, try table, perfect square or simple trinomial.

For a four term polynomial try grouping.

If none of these seven techniques work, your polynomial is prime.

Greatest Common Factor

Steps to factor a polynomial with the method of GCF (Greatest Common Factor)

1. Write out original problem.
2. Look at the coefficients and constants and find the GCF. For example if my coefficients are 4, 8 and 10 then my GCF would be 2.
3. Look at each term to see if there is a common variable. The GCF for the variable factor will be the one with lowest exponent. For example the GCF for x^2, x^3, and x^5, will be x^2.
4. The GCF for the coefficients and the GCF for the variables are placed to the left of a ( ).
5. The content of the ( ) will be the result of dividing the GCF into the original polynomial.

Example: Factor $10x^2 - 5x$ with the GCF method:

$10x^2 - 5x$

$5x(2x - 1)$

Factoring can be checked by multiplication. If we multiply $5x$ times $(2x - 1)$ we will obtain: $10x^2 - 5x$.

Factor: $12x + 13y$ (Is Prime, no common number or variable).

Factoring by Grouping

Steps to factor a four term polynomial with grouping

1. Write out original problem.
2. Try GCF.
3. If GCF works or not you will end up with four terms.
4. Look at the first two terms for a common factor and then look at the last two terms for a common factor.
5. Factor the 1st and 2nd terms using a GCF and factor the 3rd and 4th terms using a GCF.
6. You will end up with two sets of ( ) and the contents should be the same.
7. Make two sets of ( ) together like ( )( ).
8. The first set will have the common ( ) from step 6 and the second set will have the individual GCF's from step 5.
9. For the grouping method you may have to take out a minus factor or take out a factor of just one. **You can also move the four terms around to facilitate factoring by grouping**.
10. Check by multiplication.

Example: Factor $\underline{5x + 15 + xy + 3y}$ by grouping.

$5x + 15 + xy + 3y$
$5(x + 3) + y(x + 3)$
$(5 + y)(x + 3)$

Check
$(5 + y)(x + 3)$
$5x + 15 + xy + 3y$

Factoring a Trinomial with a Table

Steps to factor a trinomial using the table method

1. Write out original problem.
2. Try GCF.
3. Using original or new trinomial go off to the right side of the problem and write down ***a*** = __, ***b*** = ___ and ***c*** = ____. The values for a, b and c are coefficients from a trinomial of the form of, $\boldsymbol{a}x^2 + \boldsymbol{b}x + \boldsymbol{c}$.
4. Find the product of (a)(c) and make a table as follows:

Factors of (a)(c) = ___		Sum is b = ___ ?

5. Find all of the factor combinations of your product and then add the factors until you get the value of ***b***. Watch the signs.
6. If the product is positive, you can have two positive factors or two negative factors.
7. If the product is negative you can have a large positive and small negative factor pair or vice-versa.
8. Once you can find a sum that is equal to the value of ***b***, STOP! … and circle that row of factors.
9. If you have tried all of the factors of the product and none of the factors add up to the value of [b], then the trinomial cannot be factored any more in this problem.
10. Go back to the left and rewrite your trinomial as a four term polynomial. The old middle term of the trinomial will be replaced by two terms and their coefficients come from columns 1 & 2 of your table.
11. Factor the four term polynomial by grouping.
12. Check by multiplying.

Example: Factor $6x^2 - 11x - 10$ with the table method.

$6x^2 - 11x - 10$
$6x^2 + 4x - 15x - 10$ a = 6, b = -11, c = -10
$2x(3x + 2) - 5(3x + 2)$
$(2x - 5)(3x + 2)$

Factors of (a)(c) (6)(-10) $\boxed{-60}$		Sum is b ? $\boxed{-11}$
1	-60	-59
2	-30	-28
3	-20	-17
4	-15	-11

Factoring a Trinomial with a Leading Coefficient of One (Simple)

Steps to factor a Simple Trinomial in form of $ax^2 + bx + c$ and **<u>a = 1</u>**

1. Write out original problem.
2. Write down a = , b = , c =
3. Multiply (a)(c) which will just be the value of **[c]** because a is 1.
4. Use the table and find factor combinations of **[c]** that add together to give you **[b]**.
5. Write two sets of ( )'s: ( )( ).
6. Put x as first term in each ( ).
7. The last term in each ( ) are the factors you found in step 4. *You do not need to split trinomial into four terms*.

Example: Factor $x^2 - 3x - 18$

$x^2 - 3x - 18$

$(x + 3)(x - 6)$ $\quad$ $a = 1, b = -3, c = -18$

Factors of (a)(c) **[-18]**		Sum is b? **[-3]**
1	-18	-17
2	-9	-7
3	-6	-3

Factoring a Perfect Square Trinomial

Steps to Factor a Perfect Square Trinomial

1. Write out original problem.

2. Try GCF.

3. Look for these conditions:
   a. The coefficients of the 1st and 3rd terms are perfect squares like: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196 and 225. The first and third terms are positive.
   b. If variables exist on the 1st & 3rd terms, they must have even exponents, like x^2, x^4, x^6, etc.
   c. The middle term is either positive or negative and twice the product of the square roots of the 1st and 3rd terms.

4. In formula form where, *a* and *b* can be any expression:

   $a^2 + 2ab + b^2$, factors to:
   $(a + b)(a + b)$

   or

   $a^2 - 2ab + b^2$, factors to:
   $(a - b)(a - b)$

5. To factor, do the following:
   a. Make two sets of parenthesis like ( )( ) .
   b. The first term in each ( ) is the square root of the first term of trinomial. The last term in each ( ) is the square root of the last term in the trinomial.
   c. The sign in each ( ) is the same and will be the sign of the middle term of the trinomial.

6. Check by Multiplication.

Examples to Factor:

A) $25x^2 + 20x + 4$
$(5x + 2)(5x + 2)$

B) $25x^2 – 70x + 49$
$(5x - 7)(5x - 7)$

Factoring a Binomial with Difference of Squares

Steps to Factor a Binomial with Difference of Squares

1. Write out original problem.
2. Try GCF.
3. Look for conditions:
   a. Each term is a perfect square.
   b. There is a minus between the terms.
4. You can factor as follows:
   a. Draw two sets of ( )’s, like ( )( ) .
   b. Put + in 1st ( ) and – in 2nd ( ).
   c. 1st term in each ( ) is square root of 1st term in binomial.
   d. Last term in each ( ) is square root of last term in binomial.

Examples:

A) $9x^2 – 16y^2$
$(3x + 4y)(3x – 4y)$

B) $x^2 – 1$
$(x + 1)(x – 1)$

Steps to Factor a Binomial with Sum or Difference of Cubes

Steps to factor a binomial with sum or difference of cubes

1. Write out original problem.
2. Try GCF.
3. Look for one condition:
   - Both terms are perfect cubes, which means that coefficients could be 1, 8, 27, 64, 125, etc. and the exponents on variables are multiples of three like, x^3, x^6, x^9.
4. Draw two sets of ( )’s one set holds two terms and the second holds three terms.
5. The contents of the first ( ) are the following:
   a. The first term is the cube root of the first term of the binomial.

   b. The sign is the same as the sign of the binomial.
   c. The last term is the cube root of the last term of the binomial.
6. The contents of the second ( ) are the following:
   a. 1^{st} term is the square of the 1^{st} term of 1^{st} ( ).
   b. The sign of the 2^{nd} term is the opposite of the sign in the 1^{st} ( ).
   c. The 2^{nd} term is the product of the terms of the 1^{st} ( ).
   d. The sign of the last term is always positive.
   e. The last term is the square of the last term of the 1^{st} ( ).
7. In formula form:
   a. $(a^3 + b^3)$ factors to,
      $(a + b)(a^2 – ab + b^2)$
   b. $(a^3 – b^3)$ factors to,
      $(a – b)(a^2 + ab + b^2)$

Example: Factor $x^3 – 27$

$x^3 – 27$
$(x – 3)(x^2 + 3x + 9)$

Example: Factor $64 + y^3$

$64 + y^3$
$(4 + y)(16 – 4y + y^2)$

Factoring a Polynomial of Any Size

General Strategy for Factoring Polynomials
1. Write out original problem.
2. Try GCF
3. If GCF works, you will have a "new" polynomial in a ( ); look at this new polynomial for more factoring. If GCF does not work, check original polynomial for more factoring.
4. If there are two terms, try:
   a. Difference of Squares
   b. Sum or Difference of Cubes
5. If there are three terms, try:
   a. Table Method
   b. Simple Method
   c. Perfect Square Trinomial

NOTE: The Table Method can factor all trinomials. If you only want to memorize one method, then learn the Table Method well.

6. If there are four terms, try:

a. Grouping

7. Look at all new factors with two or more terms and try and do more factoring.
8. Check by multiplication.

Examples to Factor: **NOTE: Don't forget to look at GCF !**

$64x^2 – 100$
$4(16x^2 – 25)$ **<< Now we see a difference of squares.**
$4(4x + 5)(4x – 5)$

$4x^4y – 8x^3y – 60x^2y$
$4x^2y(x^2 – 2x – 15)$ **<< Now we see a trinomial that can be factored.**
$4x^2y(x + 3)(x – 5)$

$10x^2 – 25x – 60$
$5(2x^2 – 5x – 12)$ **<< Now we see a trinomial that can be factored.**

See table for factoring.

$5[2x^2 + 3x – 8x – 12]$ **<< It is helpful to use [ ] and not ( ).**
$5[x(2x + 3) – 4(2x + 3)]$
$5[(2x + 3)(x – 4)]$
$5(2x + 3)(x – 4)$

The [ ] can be removed on last step because they are not necessary to show multiplication.

a = 2, b = -5, c = -12

Factors of (a)(c) (2)(-12) **-24**		Sum is b ? **-5**
1	-24	-23
2	-12	-10
3	-8	-5