

## Scientific Notation

Scientific notation is used to write very large and very small numbers in a compact form. Scientific notation can also be used to simplify multiplication and division very efficiently. The standard form for a number in scientific notation is:

$$a \times 10^n,$$

where  $a$  is called the **mantissa**. The value of  $a$  is greater than or equal to one and less than 10 as shown:

$$1 \leq a < 10.$$

The value of  $n$ , the exponent, can be any integer.

The following sets of steps show how to multiply, divide and do powers with numbers in scientific notation.

## Steps to Simplify Multiplication with Scientific Notation

1. Write out original problem.
2. Put each number in scientific notation.
3. Rewrite the problem with the mantissa's in one ( ) and the powers of 10 in another ( ).
4. Multiply the numbers in the first ( ) together and use exponent rules and add the exponents in the second ( ).
5. If the new mantissa, is not,  $1 \leq a < 10$ , convert the mantissa to scientific notation and then combine the two powers of 10 together.

Example with comments

Comments	Simplification
Write the original problem and the first number, being multiplied, is in scientific notation and we convert the second number to scientific notation.	$(3 \times 10^{12})(82,000) = (3 \times 10^{12})(8.2 \times 10^4)$
The two mantissa's are placed in one ( ) and the powers of 10 in another ( ).	$= (3 \times 8.2) \times (10^{12} \times 10^4)$
We multiply the numbers in the first ( ) and in the second ( ) the exponents of are added and placed on one base of 10.	$= (24.6) \times (10^{16})$
In the first ( ) the number is greater than 10 so it has to be put in scientific notation.	$= (2.46 \times 10^1) \times (10^{16})$
Finally, the two powers of 10 are put as one power of 10.	$= 2.46 \times 10^{17}$

## Steps to Simplify Division with Scientific Notation

1. Write out original problem.
2. Put each number in scientific notation.
3. Rewrite the problem with the mantissa's in one fraction and the powers of 10 in another fraction.
4. Divide the numbers in the first fraction and exponent rules and subtract the exponents in the second fraction.
5. If the new mantissa, is not,  $1 \leq a < 10$ , convert the mantissa to scientific notation and then combine the two powers of 10 together.

Comments	Simplification
Write the original problem; both numbers are in scientific notation. The mantissa's are put in one fraction and powers of 10 in another fraction.	$\frac{4.2 \times 10^{12}}{6 \times 10^{20}} = \frac{4.2}{6} \times \frac{10^{12}}{10^{20}}$
The numbers in the first fraction are divided. The powers of 10 in second fraction are simplified by subtracting exponents.	$= 0.7 \times 10^{-8}$
The new mantissa is less than one so it has to be converted to scientific notation.	$= 7 \times 10^{-1} \times 10^{-8}$
Finally the two powers of 10 are put as one power of 10.	$= 7 \times 10^{-9}$

## Steps to Simplify a Number in Scientific Notation to a Power

1. Write out original problem.
2. Use product to power rule and place mantissa in a ( ) and place the power of 10 in a ( ). The original power is placed outside of each ( ).
3. Expand number off to side in first ( ) and put result back problem. The second ( ) is simplified by multiplying exponents.
4. If the new mantissa, is not,  $1 \leq a < 10$ , convert the mantissa to scientific notation and then combine the two powers of 10 together.

Comments	Simplification
Write the original problem; both numbers are in scientific notation. The mantissa's are put in one ( ) and powers of 10 in another ( ).	$(2.5 \times 10^4)^3 = (2.5)^3 \times (10^4)^3$ $(2.5)^3 = (2.5)(2.5)(2.5)$ $= 6.25(2.5)$ $= 15.625$
The number in the first ( ) is expanded off to the side.	$= 15.625 \times 10^{12}$
The new mantissa is greater than 10 so it has to be converted to scientific notation.	$= 1.5625 \times 10^1 \times 10^{12}$
Finally the two powers of 10 are put as one power of 10.	$= 1.5625 \times 10^{13}$

Check out the following website:

<http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/index.html>