Ratios, Rates and Proportions

5.1: A **ratio** is a comparison of two numbers. Ratios are very common in everyday life and the word, "to," is important when specifying a ratio. Frozen juices have a ratio of one can of concentrate ***to*** three cans of water.

There are three common ways to express a ratio:

1. a to b
2. $a:b$
3. $\frac{a}{b}$

The fraction form is used the most in math.

Step to Write and Reduce a Ratio

1. Write out original problem.
2. Identify the word, "**to**".
3. Make a fraction:
   a. Write the number and units to the left of the word "to" in the numerator.
   b. Write the number and units to the right of the word "to" in the denominator.
4. Follow rules to reduce fractions by using factors and also cancel the same units over the same units.

Example: Write the ratio in lowest terms of 12 in. to 18 in. Note the cancelation of the units for inches, "**in**", with a ✓ .

$$\frac{12\,\text{in}}{18\,\text{in}} = \frac{\cancel{2}\cdot 2\cdot \cancel{3}\,\cancel{\text{in}}}{\cancel{2}\cdot \cancel{3}\cdot 3\,\cancel{\text{in}}}$$

$$= \frac{2}{3}$$

5.2: A **rate** is a type of ratio that compares two <u>different</u> units. Example, Joe drove 300 miles and car used 10 gallons. The rate is: $\frac{300\,\text{mi}}{10\,\text{gal}}$.

A rate can also be reduced, but the units <u>will not cancel</u> because they are different.

$$\frac{300\,\text{mi}}{10\,\text{gal}} = \frac{3\cdot \cancel{2}\cdot 2\cdot \cancel{5}\cdot 5\,\text{mi}}{\cancel{2}\cdot \cancel{5}\,\text{gal}}$$

$$= \frac{3\cdot 2\cdot 5\,\text{mi}}{\text{gal}}$$

$$= \frac{6\cdot 5\,\text{mi}}{\text{gal}}$$

$$= \frac{30\,\text{mi}}{\text{gal}}$$

It is common for rates to be written as unit rates. A unit rate has a denominator with a number of 1.

Step to Make a Unit Rate
1. Write out original problem.
2. Set up rate as a fraction.
3. Divide number in numerator by number in denominator.
4. Units can be written with a / or with words or an abbreviation.

Example: Make a unit rate to measure the rate of driving 400 miles in 6 hours. Round off to tenths.

$$\frac{400\,\text{mi}}{6\,\text{hr}} \approx 66.7\,\frac{\text{mi}}{\text{hr}} \text{ or } 66.7\,\text{mph}$$

$$\begin{array}{r} 66.66 \\ 6\overline{)400.00} \\ -36 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 4 \end{array}$$

5.3: A **proportion** is an equation with two rates or two ratios. A proportion will have one ratio or rate with all known values and the other ratio or rate will have an unknown value. The unknown value is represented by a variable.

Example: A cookie recipe calls for 3 cups of sugar for every batch of 25 cookies. How much sugar is needed for 60 cookies?

The first sentence tells us the rate of sugar (3 cups) to a certain number of cookies (25 cookies). The other rate only tells us the number of cookies (60 cookies). We do not know the amount of sugar so we set a variable:

Let *x* = the number of cups of sugar needed to make 60 cookies.

The proportion is:

$$\frac{3\,\text{cups}}{25\,\text{cookies}} = \frac{x\,\text{cups}}{60\,\text{cookies}}$$

We now need to know how to find the value of ***x*** or we need to solve for ***x***.

Steps to Solve a Proportion for a Variable
1. Write out problem.
2. Set up proportion with one fraction equal to another fraction.
3. The units can be removed from the fractions.
4. Reduce the fraction with known values, if needed.

5. Put  over the equal sign to show a cross product.
6. The cross product is accomplished on the next step by writing an equation with parenthesis, $(\ )(\ ) = (\ )(\ )$.
7. The pair $(\ )(\ )$ on the left side of the equal sign will have the numerator of 1st fraction with the denominator of the 2nd fraction.
8. The pair $(\ )(\ )$ on the right side of the equal sign will have the numerator of 2nd fraction with the denominator of the 1st fraction.
9. On the next step, put a fraction bar under left side of equal sign and another bar under right side of equal sign. Below the fraction bar put the $(\ )$ containing the number by the variable.
10. Cancel out $(\ )$ in numerator and denominator of fraction by the variable.
11. On the other side of the equation, factor as needed, cancel like factors, multiply remaining factors.
12. There will now be just one variable on one side of the equal sign and a number on the other side.
13. If the variable is on the right of the equal sign, flip the entire equation so the variable appears on the left side.

Example: Solve $\frac{3\,\text{cups}}{25\,\text{cookies}} = \frac{x\,\text{cups}}{60\,\text{cookies}}$ for ***x***.

Comments	Steps
Write original problem.	$\frac{3\,\text{cups}}{25\,\text{cookies}} = \frac{x\,\text{cups}}{60\,\text{cookies}}$
Remove units.	$\frac{3}{25} = \frac{x}{60}$
Write cross product symbol on equal sign.	$\frac{3}{25} \times \frac{x}{60}$
Show the factors of the cross product of numerator of 1st fraction with denominator of 2nd fraction and numerator of 2nd fraction with denominator of 1st fraction.	$(3)(60) = (x)(25)$
Make a fraction on each side with the denominator being the number by the variable. In this case the number is (25).	$\frac{(3)(60)}{(25)} = \frac{(x)(25)}{(25)}$
Factor left fraction and cancel like factors on both fractions.	$\frac{(3)(2)(3)(2)\cancel{(5)}}{(5)\cancel{(5)}} = \frac{(x)\cancel{(25)}}{\cancel{(25)}}$
Write remaining factors.	$\frac{(3)(2)(3)(2)}{(5)} = \frac{(x)}{1}$
Simplify both fractions.	$\frac{6(3)(2)}{5} = x$

Comments	Steps
Continue to simplify.	$\frac{18(2)}{5} = x$
Continue to simplify.	$\frac{36}{5} = x$
Convert improper to a mixed because worded problems make more sense with mixed numbers.	$7\frac{1}{5} = x$
Flip equation around so that variable is on left side.	$x = 7\frac{1}{5}$

Write the answer with a complete sentence.

Seven and one-fifth cups of sugar are needed to make 60 cookies.