

Standard Form of a Quadratic Equation with One Variable:

$$ax^2 + bx + c = 0 ,$$

where,

$a$ ,  $b$  and  $c$  are constants and  $a \neq 0$ .

Steps to Solve Quadratic Equations by Factoring

1. Write out problem.
2. Simplify both sides of the equation.
3. Use properties of equality to put all terms on left side in standard form of,  $ax^2 + bx + c = 0$ .
4. If leading term is not positive, multiply both sides of the equation by  $-1$ .
5. Factor left side.
6. On line after factoring, take each factor with a variable and set the factor equal to zero.
7. Put the word **or** between the equations and solve each equation.
8. Check all values and if they check write, "Solution: { \_\_\_\_, \_\_\_\_ }."
9. If values do not check find errors.

Example 1: for Solve by Factoring:

$x^2 + 13x = -36$ $x^2 + 13x + 36 = -36 + 36$ $x^2 + 13x + 36 = 0$ $(x + 9)(x + 4) = 0$ $x + 9 = 0 \qquad \text{or} \qquad x + 4 = 0$ $x + 9 - 9 = 0 - 9 \qquad \text{or} \qquad x + 4 - 4 = 0 - 4$ $x = -9 \qquad \qquad \qquad \text{or} \qquad x = -4$	<p>Put all terms on left side.</p>   <p>All terms are now on left side. Now, we can factor.</p>  <p>Make "little" equations with each factor and set each factor equal to zero. We can do this based on Zero-Product Principle.</p> <p>Solve each of these little equations.</p>
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**Check:**  $x^2 + 13x = -36$  for  $x = -9$ , and  $x = -4$ .

$$\begin{aligned} (-9)^2 + 13(-9) &\stackrel{?}{=} -36 \\ 81 - 117 &\stackrel{?}{=} -36 \\ -36 &= -36 \end{aligned}$$

$$\begin{aligned} (-4)^2 + 13(-4) &\stackrel{?}{=} -36 \\ 16 - 52 &\stackrel{?}{=} -36 \\ -36 &= -36 \end{aligned}$$

**Solution:**  $\{-9, -4\}$ .

Worded problems in this chapter will be solved using quadratic equations.

**Example:** A rectangle has a length that is 3ft longer than its width. The area of the rectangle is 18ft. Find the width and length.

We will first define the variable information and start with the information, which is the most unknown. The width is the most unknown.

Let  $x$  = the width of the rectangle in ft.

Let  $x + 3$  = the length of the rectangle in ft.

(Note: Area of a rectangle is length times width so we will set up an equation.)

length      width      area  
↙      ↘      ↙  
 $(x + 3)(x) = 18$

Now we will solve the equation.

$$\begin{aligned}(x + 3)(x) &= 18 \\x^2 + 3x &= 18 \\x^2 + 3x - 18 &= 0 \\(x + 6)(x - 3) &= 0\end{aligned}$$

$$\begin{array}{ll}x + 6 = 0 & \text{or} \quad x - 3 = 0 \\x + 6 - 6 = 0 - 6 & \text{or} \quad x - 3 + 3 = 0 + 3 \\x = -6 & \text{or} \quad x = 3\end{array}$$

We are finding a dimension so the negative value is not used. We only use the positive 3.

Since  $x$  is 3, that is the width in feet, then the length is  $x + 3$  which will be 6, the length in feet.

We will summarize with a sentence.

**The width of the rectangle is three feet and the length of the rectangle is six feet.**