Steps For Adding or Subtracting Rational Expressions

1. Write out the original problem.
2. Prep all fractions to make sure numerator and denominator are in standard form and have the leading term positive.
3. If all denominators are the same, proceed to step 8.
4. Go to the side and factor each denominator on a separate row and additionally factor each monomial factor completely.
5. Stack common factors between denominators in the same column. The *LCD* is found by bringing down a factor in each column.
6. Rewrite each fraction and put each numerator in a ( ) and rewrite each denominator with factors.
7. Look at each denominator and see what factor or factors are necessary to multiply by the denominator to get the *LCD*. Once these factors are known, then insert the necessary factors in each **denominator and numerator**.
8. Make one new fraction by putting all numerators in one numerator. Insert [ ] around each old numerator when inserting and make sure you have the plus or minus signs present. The denominator is the *LCD*.
9. Simplify the numerator and rewrite the denominator with the actual factors.
10. Simplify fraction by factoring and canceling like factors.

Example follows on next page.

Simplify. $\dfrac{x+6}{x-3} - \dfrac{x}{x+2} + \dfrac{10}{x^2 - x - 6}$

Notes on Simplification:	**Simplification**
W.O.P.	$\dfrac{x+6}{x-3} - \dfrac{x+1}{x+2} + \dfrac{10}{x^2 - x - 6}$
Factor denominators on the right, stack like factors and find the LCD.	$x-3 = \quad (x-3)$ $x+2 = \quad (x+2)$ $x^2 - x - 6 = (x-3)(x+2)$ --------------- $LCD = \quad (x-3)(x+2)$
Rewrite each fraction with a factored denominator and each numerator in ( ).	$= \dfrac{(x+6)}{(x-3)} - \dfrac{(x+1)}{(x+2)} + \dfrac{(10)}{(x-3)(x+2)}$
Compare each denominator to *LCD* and see what factors are missing. These factors are inserted in both the numerator and denominator of each fraction. Notice the different colors.	$= \dfrac{(x+6)(x+2)}{(x-3)(x+2)} - \dfrac{(x+1)(x-3)}{(x+2)(x-3)} + \dfrac{(10)}{(x-3)(x+2)}$
Put each numerator in a [ ] and then insert all numerators into one fraction.	$= \dfrac{\left[(x+6)(x+2)\right] - \left[(x+1)(x-3)\right] + \left[(10)\right]}{(x-3)(x+2)}$
We now simplify the numerator and this takes many lines of algebra.	$= \dfrac{\left[x^2 + 2x + 6x + 12\right] - \left[x^2 - 3x + 1x - 3\right] + \left[10\right]}{(x-3)(x+2)}$
	$= \dfrac{\left[x^2 + 8x + 12\right] - \left[x^2 - 2x - 3\right] + \left[10\right]}{(x-3)(x+2)}$
.	$= \dfrac{x^2 + 8x + 12 - x^2 + 2x + 3 + 10}{(x-3)(x+2)}$
	$= \dfrac{x^2 - x^2 + 8x + 2x + 12 + 3 + 10}{(x-3)(x+2)}$
The numerator is now simplified. Factor numerator and check for cancelling of like factors.	$= \dfrac{10x + 25}{(x-3)(x+2)}$
We cannot cancel any factors, so this is the final form.	$= \dfrac{5(2x+5)}{(x+2)(x-3)}$

The next section deals with equations that have fractions. The steps here involve getting the denominators cleared so that we end up with a polynomial equation. At this point the equation can be solved using the techniques we have learned for solving equations to the first power or to higher powers.

Steps for solving equations with fractions (Section 4.4)

1. Write out the original problem.
2. Look at fractions and find the lowest common denominator (*LCD*). This is accomplished by factoring the denominators and then seeing what combination of factors is the lowest. Make sure each factor in the *LCD* is enclosed in a ( ).
3. Check all factors containing a variable in the *LCD* for **<u>restrictions</u>**. Restrictions are found by setting these factors $\neq$ **0** and solving. Write the restrictions by the *LCD*. Go back to each fraction and write each denominator in its factored form and put each numerator in the ( ).
4. Multiply each side of the equation by the *LCD*. We can do this by way of the Multiplicative Property of Equality which allows us to multiply each side of the equation by the same thing.
5. When multiplying each side of the equation by the *LCD*, first enclose each side with [ ].
6. Continue the multiplication process by distributing the *LCD* times each individual fraction within the [ ].
7. After distributing the *LCD* with each fraction, keep each fraction in a [ ] to insure proper use of the signs in front of each fraction.
8. Cancel out the like factors in the numerator and denominator and you will end up with just the numerators.
9. **CAUTION: It is important to be careful here to notice when you are solving for a variable in an equation and when you are just adding and subtracting fractions. DO NOT multiply through by the *LCD* unless you have an equation and are <u>solving</u>.**
10. After all of the denominators have been eliminated, simplify each side of the equation.
11. If the highest degree of the variable term is to the 1^{st} power, solve the equation by getting the variable term on the left and the constant term on the right.
12. If the highest degree is over 1 then get all terms on left side and solve by factoring or the quadratic formula.
13. Check each answer by first making sure it is not a restriction and then substitute if a formal check is needed. Write down all solutions after checking.

Example:

Solve for x: $\dfrac{4}{4x^2 - 9} - \dfrac{5}{4x^2 - 8x + 3} = \dfrac{8}{4x^2 + 4x - 3}$

Steps on next page.

Notes on Solving:	Simplification
Write out problem and notice that this is an equation and that the instructions did state to solve.	$\frac{4}{4x^2-9}-\frac{5}{4x^2-8x+3}=\frac{8}{4x^2+4x-3}$
(On the side get *LCD*).	$4x^2-9=(2x+3)(2x-3)$ $4x^2-8x+3=(2x-3)(2x-1)$ $4x^2+4x-3=(2x+3)(2x-1)$ $LCD=(2x+3)(2x-3)(2x-1)$
(On the side find the restrictions using the factors in the *LCD*.)	$2x+3\neq 0 \text{ or } 2x-3\neq 0 \text{ or } 2x-1\neq 0$ $2x\neq -3 \text{ or } 2x\neq 3 \text{ or } 2x\neq 1$ $\frac{2x}{2}\neq -\frac{3}{2} \text{ or } \frac{2x}{2}\neq\frac{3}{2} \text{ or } \frac{2x}{2}\neq\frac{1}{2}$ $x\neq -\frac{3}{2} \text{ or } x\neq\frac{3}{2} \text{ or } x\neq\frac{1}{2}$ **Restrictions:** $x\neq -\frac{3}{2} \text{ or } x\neq\frac{3}{2} \text{ or } x\neq\frac{1}{2}$
Rewrite each fraction with a factored denominator and each numerator in ( ).	$\frac{(4)}{(2x+3)(2x-3)}-\frac{(5)}{(2x-3)(2x-1)}=\frac{(8)}{(2x+3)(2x-1)}$
Put each side of the fraction in a [ ] and then multiply each side by the *LCD*.	$(2x+3)(2x-3)(2x-1)\left[\frac{(4)}{(2x+3)(2x-3)}-\frac{(5)}{(2x-3)(2x-1)}\right]=(2x+3)(2x-3)(2x-1)\left[\frac{(8)}{(2x+3)(2x-1)}\right]$
Distribute the *LCD* to each fraction in [ ], keep a [ ] around each fraction. Cancel like factors.	$\left[\frac{\cancel{(2x+3)}\cancel{(2x-3)}(2x-1)(4)}{\cancel{(2x+3)}\cancel{(2x-3)}}\right]-\left[\frac{(2x+3)\cancel{(2x-3)}\cancel{(2x-1)}(5)}{\cancel{(2x-3)}\cancel{(2x-1)}}\right]=\left[\frac{\cancel{(2x+3)}(2x-3)\cancel{(2x-1)}(8)}{\cancel{(2x+3)}\cancel{(2x-1)}}\right]$
All denominators cancel, continue to solve equation.	$[(2x-1)(4)]-[(2x+3)(5)]=[(2x-3)(8)]$ $[8x-4]-[10x+15]=[16x-24]$
The equation is a 1st order equation so we will work on getting the variable terms on the left and the constant terms on the right.	$8x-4-10x-15=16x-24$ $8x-10x-4-15=16x-24$ $-2x-19=16x-24$ $-2x-19+19=16x-24+19$ $-2x=16x-5$ $-2x-16x=16x-16x-5$ $-18x=-5$ $\frac{-18x}{-18}=\frac{-5}{-18}$
The answer is $x=\frac{5}{18}$ and this is not a restriction so it is a valid solution if it checks.	$x=\frac{5}{18}$

Note: Check is very involved, notes will be given later.

We now will check the answer by substituting $x = \frac{5}{18}$ in to the original equation.

Check:

$$\frac{4}{4x^2 - 9} - \frac{5}{4x^2 - 8x + 3} = \frac{8}{4x^2 + 4x - 3}$$

let $x = \frac{5}{18}$

$$\frac{4}{4\left(\frac{5}{18}\right)^2 - 9} - \frac{5}{4\left(\frac{5}{18}\right)^2 - 8\left(\frac{5}{18}\right) + 3} = \frac{8}{4\left(\frac{5}{18}\right)^2 + 4\left(\frac{5}{18}\right) - 3}$$

$$\frac{4}{4\left(\frac{25}{324}\right) - 9} - \frac{5}{4\left(\frac{25}{324}\right) - 8\left(\frac{5}{18}\right) + 3} = \frac{8}{4\left(\frac{25}{324}\right) + 4\left(\frac{5}{18}\right) - 3}$$

$$\frac{4}{\frac{100}{324} - 9} - \frac{5}{\frac{100}{324} - \frac{40}{18} + 3} = \frac{8}{\frac{100}{324} + \frac{20}{18} - 3}$$

$$\frac{4}{\frac{25}{81} - 9} - \frac{5}{\frac{25}{81} - \frac{20}{9} + 3} = \frac{8}{\frac{25}{81} + \frac{10}{9} - 3}$$

$$\frac{4}{\frac{25}{81} - \frac{9}{1}} - \frac{5}{\frac{25}{81} - \frac{20}{9} + \frac{3}{1}} = \frac{8}{\frac{25}{81} + \frac{10}{9} - \frac{3}{1}}$$

$$\frac{4}{\frac{25}{81} - \frac{9(81)}{1(81)}} - \frac{5}{\frac{25}{81} - \frac{20(9)}{9(9)} + \frac{3(81)}{1(81)}} = \frac{8}{\frac{25}{81} + \frac{10(9)}{9(9)} - \frac{3(81)}{1(81)}}$$

$$\frac{4}{\frac{25}{81} - \frac{729}{81}} - \frac{5}{\frac{25}{81} - \frac{180}{81} + \frac{243}{81}} = \frac{8}{\frac{25}{81} + \frac{90}{81} - \frac{243}{81}}$$

$$\frac{4}{\frac{-704}{81}} - \frac{5}{\frac{88}{81}} = \frac{8}{\frac{-128}{81}}$$

$$4\left(\frac{-81}{704}\right) - 5\left(\frac{81}{88}\right) = 8\left(\frac{-81}{128}\right)$$

$$\frac{-81}{176} - \frac{405}{88} = -\frac{81}{16}$$

$$\frac{-81}{176} - \frac{405(2)}{88(2)} = -\frac{81}{16}$$

$$\frac{-81}{176} - \frac{810}{176} = -\frac{81}{16}$$

$$-\frac{891}{176} = -\frac{81}{16}$$

$-\frac{81}{16}=-\frac{81}{16}$ ✓

The next section deals with solving literal equations [equations with more than one variable] or formulas for a particular variable in the equation. We have done this to some extent with equations involving both x and y. Recall that we would solve the equation for x or for y.

Steps for solving literal equations (Section 4.4)

1. Write out the original problem.
2. Write down the variable that being solved.
3. Simplify each side of the equation completely. Common operations are multiplying out ( )'s. Using the distributive property, collecting like terms, etc.
4. If there are fractions in the problem, find the lowest common denominator (*LCD*) and multiply each side of the equation by the *LCD*.
5. After each is simplified add or subtract terms from each side so that all terms that have the variable that is being solved for end up on the left side and all other terms end up on the right side. This is similar to solving an equation for a variable except that there will be multiple terms containing the variable being solved and the other side will not have constants but will have a variety of polynomial terms.
6. Simplify each side as much as possible by combing like terms.
7. The left side will now have one term or may have more than one term. If there is more than one term than factor out the variable out of each term using the techniques learned in factoring out a greatest common factor. You will only factor out the variable and not the greatest common factor. You will now have the variable being solved times terms inside a ( ).
8. If the left side just has one term, factor the one term so that the variable being solved is outside of a ( ) and the other pieces of the term are inside of the ( ).
9. Divide each side by the term or terms inside of the ( ).
10. You will end up with the variable = to some other term or terms containing variables.
11. Simplify fraction on the left by reducing it.

Example: Solve $(3x-2)(2y-1)=b$ for y

Notes on Simplification:	**Simplification**
Write out problem and write down what variable is solved.	$(3x-2)(2y-1)=b$ Solve for y
You can simplify the left side by multiplying out the ( )'s.	$6xy-3x+4y+2=b$
Next, get all of the terms with *y* on the left side and all other terms on the right side, simplify each side.	$6xy-3x+3x+4y+2-2=b+3x-2$
You should notice there are now two terms with y in them on the left so factor out a *y*.	$6xy+4y=b+3x-2$ $y(6x+4)=b+3x-2$
Now divide each side by the terms inside of the ( ).	$\frac{y(6x+4)}{(6x+4)}=\frac{b+3x-2}{(6x+4)}$

Notes on Simplification:	Simplification
We end up with y = a fraction containing various terms.	$y = \frac{b+3x-2}{(6x+4)}$
Now reduce the fraction on the right by factoring the denominator. NOTE: The numerator can not be factored.	$y = \frac{b+3x-2}{2(3x+2)}$

The next sections deals with fractions when there is a mixture of operations or when there are complex fractions. Complex fractions are where there are multiple fractions within a fraction.

Steps for dealing with mixed operations and complex fractions (Section 4.5)

1. Write out the original problem.
2. If there are a variety of operations to simplify, carefully follow the order of operations. Recall this is Parenthesis, Exponents, Multiplication or Division, and finally Addition or Subtraction.
3. For example if there is a ( ) do all work within the ( ).
4. If the fraction is complex, that is it has fractions in the numerator and the denominator, combine all of the fractions in the numerator to one fraction and combine all of the fractions in the denominator to one fraction.
5. After there is just one fraction in the numerator and just one fraction in the denominator (that is for complex fractions see Step 4.) then make the problem a division problem by writing the fraction in the numerator followed by a division sign $\div$ and then followed by the fraction in the denominator. You can then simplify this division problem by, **Steps for multiplying fractions containing polynomials**.
6. If there are ( ) in the original problem or if there are complex fractions, the problem eventually gets to the point of two fractions separated by a multiplication, division, addition or subtraction sign. Follow the appropriate rules.

Example: Simplify $\frac{\frac{y+4}{y+2} - \frac{y}{y-2}}{4}$

Notes on Simplification:	Simplification
Write out problem and notice that this is a complex fraction since there are fractions in the numerator.	$\frac{\frac{y+4}{y+2} - \frac{y}{y-2}}{4}$
We will first add the fractions in the numerator to come up with one fraction. We will get a common denominator of $(y+2)(y-2)$.	$\frac{\frac{(y+4)(y-2)}{(y+2)(y-2)} - \frac{y(y+2)}{(y-2)(y+2)}}{4}$

Notes on Simplification:	Simplification
	$\dfrac{\dfrac{[(y+4)(y-2)]-[y(y+2)]}{(y-2)(y+2)}}{4}$
	$\dfrac{\dfrac{[y^2+2y-8]-[y^2+2y]}{(y-2)(y+2)}}{4}$
	$\dfrac{\dfrac{y^2+2y-8-y^2-2y}{(y-2)(y+2)}}{4}$
The numerator is now simplified to be just one fraction so now we will rewrite the problem as a division problem.	$\dfrac{\dfrac{-8}{(y-2)(y+2)}}{4}$
Now we take the reciprocal of fraction after the division sign and then change the problem to a multiplication problem.	$\dfrac{-8}{(y-2)(y+2)} \div 4$
	$\dfrac{-8}{(y-2)(y+2)} \cdot \dfrac{1}{4}$
	$-\dfrac{8}{(y-2)(y+2)(4)}$
	$-\dfrac{(2)(\cancel{4})}{(y-2)(y+2)(\cancel{4})}$
	$-\dfrac{2}{(y-2)(y+2)}$